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Estimation of the frictional strength of faults from inversion of fault-slip data: a new method

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Abstract—The conventional stress inversion methods estimate only four of the six independent parameters of the tectonic stress tensor. Using the Coulomb–Navier failure criterion as an additional constraint, it is possible to estimate the fifth parameter, characterized by the normalized critical stress difference, i.e. the critical stress difference divided by the effective overburden pressure. This parameter is related to the average friction coefficient of faults. If the stress field is uniform, faults with different orientations and at different depths have the same normalized critical stress difference. On this basis, a new method is proposed to estimate the average friction coefficient and the normalized critical stress difference from inversion of a population of faults of measured orientations and slip directions. This method is applicable both to newly formed faults and to reactivated faults. This method is applied to four data sets. In three cases, an average friction coefficient $\bar{\mu}_0 = 0.64$, 0.70 and 0.88 is obtained. One case shows a relatively low average friction coefficient $\bar{\mu}_0 = 0.22$, but this value is of poor quality due to the effect of a possibly nonuniform stress field. These results are in agreement with the average value of friction coefficient $\bar{\mu}_0 = 0.75$ derived from laboratory experiments.

INTRODUCTION

The orientations of fault planes and associated slip directions are a reflection of the state of stress in the crust. Under the assumptions that the slip direction coincides with the direction of maximum shear stress on the fault plane (Bott 1959) and that the tectonic stress field in a region is uniform, Carey & Brunier (1974) proposed a method to estimate the tectonic stress field by inversion of a population of faults. This method, variously modified and improved (Angelier 1979, Etchecopar 1981, Angelier *et al.* 1982, Angelier 1984, Gephart & Forsyth 1984, Michael 1984, Gephart 1985, Angelier 1990, Yin & Ranalli 1993), has now become a standard technique to determine the paleostress field (using fault measurements) and the contemporary stress field (using earthquake focal mechanisms) (Vasseur *et al.* 1983, Gephart & Forsyth 1984, 1985, Michael 1984, Carey-Gailhardis & Mercier 1987, Lana & Correig 1987, Michael 1987, Huang & Angelier 1989, Zoback 1989, Bergerat *et al.* 1990, Caldentey & Lana 1990, Vetter 1990, Fleischmann & Nemcok 1991, Will & Powell 1991, Wyss *et al.* 1992).

Using the above inversion method based on the geometrical constraint (i.e. the constraint imposed by fault plane orientations and slip directions), four of the six independent parameters of the tectonic stress tensor can be obtained, that is, the three principal stress directions and the stress ratio $\delta = (\sigma_2 - \sigma_3)/(\sigma_1 - \sigma_3)$ (where σ_1, σ_2, σ_3 are the principal stresses). The other two parameters (the magnitudes of two of the three principal stresses) which are related to the frictional strength and the pore fluid pressure cannot be determined on the basis of the geometrical constraint alone. However, the Coulomb–Navier failure criterion can be used as an additional constraint (i.e. the mechanical constraint) to determine the frictional strength of faults, and consequently to obtain the fifth parameter of the tectonic stress tensor (Reches 1987, Célérier 1988, Gephart 1988, Angelier 1989, Gephart 1992, Reches *et al.* 1992).

A method based on the mechanical constraint was proposed by Reches *et al.* (1992), and applied to more than 20 areas of different tectonic settings. An average friction coefficient was obtained for each area, ranging from 0.0 to 1.3 (Reches *et al.* 1992). The range and dispersion of the estimated friction coefficients are much larger than those derived from rock friction experiments (Byerlee 1978). Moreover, a zero friction coefficient implies that faulting can occur along a pre-existing fault plane subject to zero or very small tectonic shear stress. Neither earthquake observations nor *in situ* stress measurements agree with this inference (Raleigh *et al.* 1972, Hanks 1977, Zobak & Healy 1984). Apparently, the results may be related to some underlying assumption or approximation in the method, rather than to the real strength of faults.

In this paper, we first examine the basis and implications of the previous method proposed by Reches *et al.* (1992) and discuss its applicability to stress inversion. Then, we propose a new method that incorporates both the geometrical constraint and the mechanical constraint to estimate the frictional strength of geological faults from inversion of fault-slip data.

PREVIOUS INVERSION METHOD (RECHES *et al.* 1992)

Throughout the discussion, we shall refer to two Cartesian systems, x_i ($i = 1, 2, 3$) coincident with the

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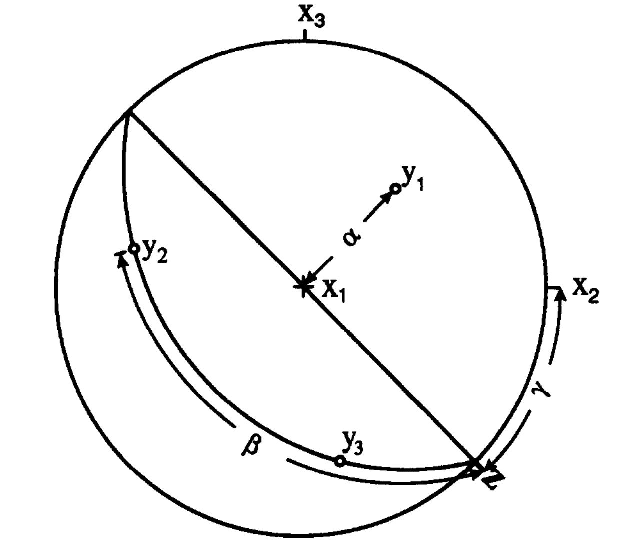

Fig. 1. Cartesian coordinate system x_i denotes the geographical coordinate system with x_1, x_2 and x_3 coinciding with the vertical, the east and the north, and y_i denotes the principal stress coordinate system with y_1, y_2 and y_3 coinciding with the σ_1, σ_2 and σ_3-axis. The unit vector **z** denoting the direction of the intersection between the y_2–y_3 plane and x_2–x_3 plane plots at point **z**. The Euler angles are defined as: α the angle between y_1 and x_1; β the angle between y_2 and **z**, measured anticlockwise on the y_2–y_3 plane starting from y_2; γ the angle between x_2 and **z**, measured clockwise from x_2 on the x_2–x_3 plane. Upper hemisphere Schmidt projection.

geographic coordinate system, and y_i coincident with the principal stress coordinate system, respectively. The transformation between x_i and y_i can be achieved by three successive rotations of the Euler angles α, β, γ (see Fig. 1); the transformation matrix is (Yin & Ranalli 1993)

$$\begin{pmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha\sin\gamma & \sin\alpha\cos\gamma \\ \sin\alpha\sin\beta & \cos\beta\cos\gamma - \cos\alpha\sin\beta\sin\gamma & -\cos\beta\sin\gamma - \cos\alpha\sin\beta\cos\gamma \\ -\sin\alpha\cos\beta & \sin\beta\cos\gamma + \cos\alpha\cos\beta\sin\gamma & -\sin\beta\sin\gamma + \cos\alpha\cos\beta\cos\gamma \end{pmatrix} \quad (1)$$

where u_i, v_i, w_i are the cosines of the angles between y_1 and x_i, y_2 and x_i, and y_3 and x_i, respectively.

In Reches' *et al.* (1992) model, a local stress tensor is associated with each fault (the unit vectors **P***, **B*** and **T*** to denote the maximum, the intermediate, and the minimum principal stress direction of this stress tensor). It is defined as follows: the **P***–**T*** plane coincides with the plane containing the slip **p** and the normal **n** to the fault plane, and **P*** makes an angle ψ^* with **p** and $\psi^* + 90°$ with **n**, where $\psi^* = (1/2)\tan^{-1}(1/\mu_0)$, μ_0 being the friction coefficient. Consequently, the maximum local shear stress coincides with the slip direction on the fault plane, and the critical stress difference is minimized for each fault according to the Coulomb–Navier failure criterion. Reches' *et al.* (1992) method consists of searching for a general uniform stress field and an average friction coefficient for an ensemble of faults, by minimizing the sum of the angles between the principal axes of the general stress tensor and the corresponding principal axes of the local stress tensor for each fault. The criterion for this method can be written as (Reches *et al.* 1992)

$$MC_1 = \sum_{i=1}^{n} (\mathbf{P}_i^* \wedge y_1 + \mathbf{B}_i^* \wedge y_2 + \mathbf{T}_i^* \wedge y_3), \quad (2)$$

where i and $\wedge$ denote the ith fault and the angle between the two axes, respectively. Reches *et al.* (1992) also proposed another modified criterion

$$MC_2 = \sum_{i=1}^{n} [(1-\delta)\mathbf{P}_i^* \wedge y_1 + \delta\mathbf{T}_i^* \wedge y_3], \quad (3)$$

where δ is the stress ratio. In equations (2) and (3), we omit the constant denominators $3n$ and $2n$ (n is the total number of faults) which appeared in Reches' *et al.* (1992) original formulae.

Now we examine these two criteria. Since the intermediate principal stress direction is $\mathbf{B}^* = \mathbf{n} \times \mathbf{p}$, the unit vector **B*** in the x_i-cordinate system can be expressed as

$$\begin{pmatrix} B_{1x}^* \\ B_{2x}^* \\ B_{3x}^* \end{pmatrix} = \begin{pmatrix} n_{2x}p_{3x} - n_{3x}p_{2x} \\ n_{3x}p_{1x} - n_{1x}p_{3x} \\ n_{1x}p_{2x} - n_{2x}p_{1x} \end{pmatrix}, \quad (4)$$

where n_{ix} and p_{ix} are the direction cosines of **n** and **p**, respectively. The unit vector **P***, indicating the maximum principal direction of the local stress tensor, is related to **p**, **n** and **B*** as

$$\begin{aligned} P_{1x}^*p_{1x} + P_{2x}^*p_{2x} + P_{3x}^*p_{3x} &= \cos\psi^* \\ P_{1x}^*n_{1x} + P_{2x}^*n_{2x} + P_{3x}^*n_{3x} &= -\sin\psi^* \\ P_{1x}^*B_{1x}^* + P_{2x}^*B_{2x}^* + P_{3x}^*B_{3x}^* &= 0. \end{aligned} \quad (5)$$

Substituting equation (4) in (5), the components of **P*** in the x_i-coordinate system can be written as

$$\begin{aligned} P_{1x}^* &= p_{1x}\cos\psi^* - n_{1x}\sin\psi^* \\ P_{2x}^* &= p_{2x}\cos\psi^* - n_{2x}\sin\psi^* \\ P_{3x}^* &= p_{3x}\cos\psi^* - n_{3x}\sin\psi^*. \end{aligned} \quad (6)$$

Similarly, the components of **T*** in the x_i-coordinate system can be derived by vector multiplication of **B*** and **P***

$$\begin{aligned} T_{1x}^* &= -p_{1x}\sin\psi^* - n_{1x}\cos\psi^* \\ T_{2x}^* &= -p_{2x}\sin\psi^* - n_{2x}\cos\psi^* \\ T_{3x}^* &= -p_{3x}\sin\psi^* - n_{3x}\cos\psi^*. \end{aligned} \quad (7)$$

Using the transformation matrix (equation 1), the angles $\mathbf{P}^* \wedge y_1$, $\mathbf{B}^* \wedge y_2$, and $\mathbf{T}^* \wedge y_3$ which constitute the criteria MC_1 and MC_2 (see equations 2 and 3) are

$$\begin{aligned} \mathbf{P}^* \wedge y_1 &= \cos^{-1}|\cos\psi^*(u_1p_{1x} + u_2p_{2x} + u_3p_{3x}) \\ &\quad - \sin\psi^*(u_1n_{1x} + u_2n_{2x} + u_3n_{3x})| \\ \mathbf{B}^* \wedge y_2 &= \cos^{-1}|v_1(n_{2x}p_{3x} - n_{3x}p_{2x}) \\ &\quad + v_2(n_{3x}p_{1x} - n_{1x}p_{3x}) + v_3(n_{1x}p_{2x} - n_{2x}p_{1x})| \quad (8) \\ \mathbf{T}^* \wedge y_3 &= \cos^{-1}|-\sin\psi^*(w_1p_{1x} + w_2p_{2x} + w_3p_{3x}) \\ &\quad - \cos\psi^*(w_1n_{1x} + w_2n_{2x} + w_3n_{3x})|. \end{aligned}$$

The absolute value sign denotes the fact that each angle in criteria MC_1 and MC_2 refers to the acute angle between two stress axes.

From equation (8), one can see that criterion MC_1 does not constrain the stress ratio δ, because the angles $\mathbf{P}^* \wedge y_1$, $\mathbf{B}^* \wedge y_2$ and $\mathbf{T}^* \wedge y_3$ are independent of δ. Similarly, it can be proven that criterion $MC2$ does not constrain δ either, although δ appears explicitly in the equation (3). Criterion $MC2$ can be rewritten as (noting that δ is independent of $\mathbf{P}^* \wedge y1$ and $\mathbf{T}^* \wedge y3$)

$$MC_2 = \sum_{i=1}^{n} \mathbf{P}_i^* \wedge y_1 + \delta \sum_{i=1}^{n} (\mathbf{T}_i^* \wedge y_3 - \mathbf{P}_i^* \wedge y_1). \quad (9)$$

Equation (9) shows that MC_2 is a linear function of the stress ratio δ, with a slope equal to $\Sigma(\mathbf{T}_i^* \wedge y_3 - \mathbf{P}_i^* \wedge y_1)$. Therefore, MC_2 can be minimized only when $\delta = 0$ or 1 [depending on the quantity $\Sigma(\mathbf{T}_i^* \wedge y_3 - \mathbf{P}_i^* \wedge y_1)$].

The above analysis shows that criteria MC_1 and MC_2, as proposed by Reches *et al.* (1992), can be used to determine the three principal stress directions, but not the stress ratio δ. However, the principal stress directions estimated by MC_1 and MC_2 are less accurate than the methods based on the geometrical constraint (e.g. Angelier 1984, Gephart 1990, Yin & Ranalli 1993), because the slip direction on each fault is determined not only by the principal stress directions, but also by the stress ratio δ. This is reflected in Reches' *et al.* (1992) inversion result that the average misfit angle between shear stress and slip direction is large for the most data sets.

One of the main purposes of Reches' method is to determine the coefficient of friction of faults. Now we examine how much the friction coefficient is constrained by criteria MC_1 and MC_2. For simplicity, we consider a special case, i.e. $\mathbf{B}^* \wedge y_2 = 0$ for each fault. Let ψ denote the angle between the maximum principal stress direction (y_1) and the fault plane. The angles $y_1 \wedge \mathbf{p}$, $y_1 \wedge \mathbf{n}$, $y_3 \wedge \mathbf{p}$, and $y_3 \wedge \mathbf{n}$ are therefore equal to ψ, $90° + \psi$, $90° + \psi$, and $180° - \psi$, respectively. Substituting these parameters in equation (8) yields

$$\begin{aligned} \mathbf{P}^* \wedge y_1 &= \cos^{-1} |\cos\psi^* \cos\psi + \sin\psi^* \sin\psi| \\ \mathbf{B}^* \wedge y_2 &= 0 \qquad (10) \\ \mathbf{T}^* \wedge y_3 &= \cos^{-1} |\sin\psi^* \sin\psi + \cos\psi^* \cos\psi|. \end{aligned}$$

It is reasonable to assume that $\psi \leq 90°$ for each fault, i.e. the maximum shear stress direction does not lie opposite to the slip direction ($\mathbf{p}$). Therefore, the absolute value signs in equation (10) can be removed, and substituting equation (10) in equation (2) gives

$$MC_1 = 2 \sum_{i=1}^{n} \cos^{-1}(\cos\psi^* \cos\psi_i + \sin\psi^* \sin\psi_i)$$

$$= 2n\psi^* - 2 \sum_{i=1}^{n} \psi_i. \quad (11)$$

The expression for criterion MC_2 is similar to equation (11), i.e. $MC_2 = n\psi^* - \Sigma\psi_i$.

Consequently, both MC_1 and MC_2 are minimized when $\psi^* = (1/n)\Sigma\psi_i$, that is, when ψ^* is equal to the average value of the angle ψ_i between the maximum principal stress direction y_1 and each fault plane. From the relation $\psi^* = (1/2)\tan^{-1}(1/\mu_0)$, the friction coefficient μ_0 decreases with increasing $(1/n)\Sigma\psi_i$. Thus, for $(1/n)\Sigma\psi_i < 45°$, $\mu_0 > 0$ is obtained; for $(1/n)\Sigma\psi_i \geq 45°$, $\mu_0 \leq 0$. The average friction coefficient determined by criteria MC_1 and MC_2 is closely related to the average value of the angle between the maximum principal stress axis and each fault plane. This conclusion, although derived in closed form for the special case where the intermediate axes of the local and the general stress tensor coincide, can be extended to the general case where $\mathbf{B}^* \wedge y \neq 0$ by numerical simulation.

The above analysis shows that Reches' *et al.* method is applicable only to new faults in homogeneous and isotropic rocks, whose orientation with respect to the principal stress axes is determined by the friction coefficient alone. For the reactivation of preexisting faults, there is no simple relation between the fault plane orientation and the friction coefficient, because the reactivated faults are often not the most favourably oriented faults (see e.g. Ivins *et al.* 1990, Yin & Ranalli, 1992). Therefore, the 'friction coefficient' obtained from criteria MC_1 and MC_2 reflects the average value of the angles between the maximum principal stress axis and the fault planes rather than the frictional strength of faults.

A NEW INVERSION METHOD

The Coulomb–Navier failure criterion is adequate to describe both the formation of new faults and the reactivation of pre-existing faults. Yin & Ranalli (1992) have derived an expression for the critical stress difference for arbitrarily oriented pre-existing faults as a function of material parameters, depth, principal stress directions, stress ratio and orientation of fault planes

$$(\sigma_1 - \sigma_1) = \frac{\mu_0 \rho g z(1-\lambda) + S_0}{[(n_{1y}^2 + \delta^2 n_{2y}^2) - (n_{1y}^2 + \delta n_{2y}^2)^2]^{1/2} + \mu_0[(u_1^2 + \delta v_1^2) - (n_{1y}^2 + \delta n_{2y}^2)]} \quad (12)$$

where μ_o and S_0 denote the friction coefficient and the cohesion, respectively, ρ the average density of overlying rocks, g gravity, z depth, λ the pore fluid factor, n_{1y} and n_{2y} the components of the unit normal ($\mathbf{n}$) to the fault plane in the y_i-coordinate system, δ the stress ratio, and u_1 and v_1, given in equation (1), are the elements of the transformation matrix. On the basis of the critical stress difference, we propose here a new method to determine the average friction coefficient of preexisting faults from inversion of fault-slip data. Inversion techniques based on the critical stress difference have been proposed also by Angelier (1989) and Gephart (1992). However, introducing the normalized stress difference (equation 13) into an inversion criterion is new. Our method also differs from Angelier's (1989) and Gephart's (1992) methods in the formulation of inversion criteria.

Inversion criterion for the average friction coefficient

As shown by equation (12), the critical stress difference depends on depth and pore fluid pressure. However, all faults can be brought to the same normalized stress tensor ($\sigma_{1(n)}$, $\sigma_{2(n)}$, $\sigma_{3(n)}$) by dividing the magnitudes of the principal stresses σ_1, σ_2 and σ_3 by the effective overburden pressure $\rho g z(1-\lambda)$. If the stress field is uniform, all faults have the same normalized stress difference. From equation (12), and neglecting cohesion, the normalized critical stress difference is given by

$$(\sigma_{1(n)} - \sigma_{3(n)}) = \frac{(\sigma_1 - \sigma_3)}{\rho g z(1-\lambda)} \simeq \frac{\mu_0}{[(n_{1y}^2 + \delta^2 n_{2y}^2) - (n_{1y}^2 + \delta n_{2y}^2)^2]^{1/2} + \mu_0[(u_1^2 + \delta v_1^2) - (n_{1y}^2 + \delta n_{2y}^2)]}. \tag{13}$$

[The neglect of cohesion simplifies the argument and is justified by Byerlee's (1978) experimental results showing $S_0 \simeq 0$ for effective normal stress less than 200 MPa, a condition that applies to most of the crust if the pore fluid pressure is hydrostatic or larger.]

Given a set of faults, the principal stress directions and the stress ratio can be obtained from inversion based on the geometrical constraint, such as the method proposed by Yin & Ranalli (1993). Under the assumption of a uniform stress field, the average friction coefficient can also be determined by inversion. Suppose that $\bar{\mu}_0$ is the average friction coefficient and that the dispersion of the friction coefficient from the average value is small for each fault. The average normalized critical stress difference can be expressed as

$$(\bar{\sigma}_{1(n)} - \bar{\sigma}_{3(n)}) = \frac{1}{n}\sum_{i=1}^{n} (\sigma_{1(n)} - \sigma_{3(n)})_i, \tag{14}$$

where the subscript i denotes the ith fault, n the total number of faults, and $(\sigma_{1(n)} - \sigma_{3(n)})$ is given by equation (13) with replacement of μ_0 by the constant average friction coefficient $\bar{\mu}_0$ for each fault. The misfit between normalized critical stress difference and average normalized critical stress difference is

$$\Delta(\sigma_{1(n)} - \sigma_{3(n)}) = (\sigma_{1(n)} - \sigma_{3(n)}) - (\bar{\sigma}_{1(n)} - \bar{\sigma}_{3(n)}). \tag{15}$$

If the stress field is uniform and each fault has the same frictional properties, the ideal solution for the average friction coefficient should make the sum of the absolute values of misfits zero (see Fig. 2)

$$\sum_{i=1}^{n} |\Delta(\sigma_{1(n)} - \sigma_{3(n)})_i| = 0. \tag{16}$$

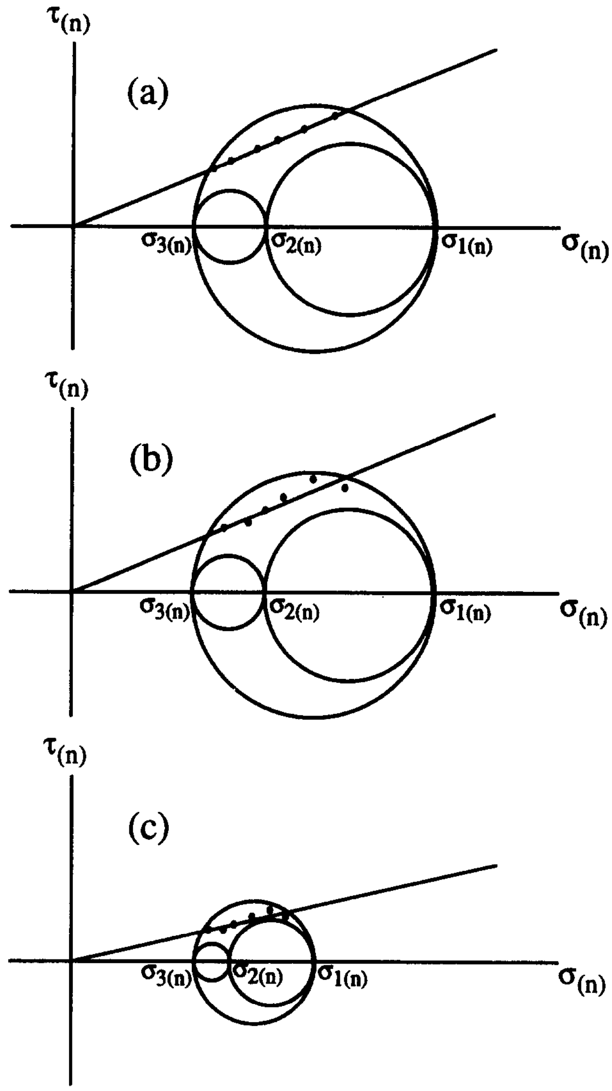

Fig. 2. Simulated orientations of faults with respect to the normalized principal stress axes. (a) The ideal case where all faults have the same average normalized critical stress difference; (b) & (c) denote the cases where there is a dispersion in the average normalized critical stress difference, and the distribution of fault orientations indicates a large average friction coefficient for (b) and a small one for (c). The average friction coefficient and the average normalized critical stress difference are given by the slope of the straight lines and the largest Mohr circles. A stress ratio $\delta = 0.3$ is assumed.

Equation (16) is usually not satisfied, because different faults may have different friction coefficients. Moreover, the misfit stress difference $\Delta(\sigma_{1(n)} - \sigma_{3(n)})$ results also from other sources, such as measurement errors, uncertainties in the three principal directions and stress ratio δ, and local stress concentrations. In other words, $\Delta(\sigma_{1(n)} - \sigma_{3(n)})$ is a random variable. From a probabilistic point of view, an inversion criterion should be devised on the basis of its probability distribution. However, it is difficult to evaluate this distribution because many of the above mentioned factors (especially the variation of the coefficient of friction and disturbances of local stresses) are unknown. Therefore, we use the method of least squares, which is widely used as an empirical criterion (Blom 1989, pp. 201–202).

Equation (13) shows that the normalized critical stress difference is an increasing function of the friction coefficient (μ_0) and it goes to zero when μ_0 is equal to zero. Consequently, the search for the average friction coefficient ($\bar{\mu}_0$) cannot be based on minimization of the sum of squares of misfit stress differences. To overcome this difficulty, we introduce a parameter $\Delta\sigma_i$, the misfit ratio for the ith fault,

$$\Delta\sigma_i = \frac{\Delta(\sigma_{1(n)} - \sigma_{3(n)})_i}{(\bar{\sigma}_{1(n)} - \bar{\sigma}_{3(n)})}, \tag{17}$$

where misfit stress difference and average normalized stress difference are given by equations (15) and (14), respectively. An inversion criterion based on minimizing the sum of squares of the misfit ratios can be expressed as

$$MC_3 = \sum_{i=1}^{n} \Delta\sigma_i^2, \tag{18}$$

The inversion consists of a search for the best solution for the average friction coefficient $\bar{\mu}_0$ by minimizing criterion MC_3. The basic physical assumption behind this inversion method is that the normalized critical stress difference at faulting is similar for each fault.

Inversion procedure and confidence intervals

Given a fault population generated by an unknown stress field, we can determine principal stress directions and stress ratio from the geometrical constraint, and average friction coefficient from the mechanical constraint. Our inversion procedure is as follows: (1) Yin & Ranalli's (1993) criterion C_6 is used to determine the first four parameters, and the hypothesis that the data set represents a uniform stress field is tested according to a procedure proposed therein. (2) A wide range of values (0.01–1.5) for the average friction coefficient (invariant for each fault) is chosen. (3) In each step of the iteration, the selected average friction coefficient is changed and the value of MC_3 is calculated according to equation (18); the increment of the selected average friction coefficient in each iterative step is 0.01. (4) The best estimate for the average friction coefficient is that value which minimizes MC_3.

Substituting the inverted average coefficient of friction in equations (13) and (14) yields the average normalized critical stress difference $(\bar{\sigma}_{1(n)} - \bar{\sigma}_{3(n)})$, from which the three average normalized principal stresses can be obtained. The principal stresses are related to the effective overburden pressure as (Yin & Ranalli 1992)

$$\rho g z(1-\lambda) = \sigma_3 + (u_1^2 + \delta v_1^2)(\sigma_1 - \sigma_3). \tag{19}$$

Using the relation $\sigma_2 = \sigma_3 + \delta(\sigma_1 - \sigma_3)$, the three average normalized principal stresses are

$$\begin{aligned}
\bar{\sigma}_{1(n)} &= \frac{\bar{\sigma}_1}{\rho g z(1-\lambda)} = 1 - (\bar{\sigma}_{1(n)} - \bar{\sigma}_{3(n)})(u_1^2 + \delta v_1^2 - 1)\\
\bar{\sigma}_{2(n)} &= \frac{\bar{\sigma}_2}{\rho g z(1-\lambda)} = 1 - (\bar{\sigma}_{1(n)} - \bar{\sigma}_{3(n)})(u_1^2 + \delta v_1^2 - \delta)\\
\bar{\sigma}_{3(n)} &= \frac{\bar{\sigma}_3}{\rho g z(1-\lambda)} = 1 - (\bar{\sigma}_{1(n)} - \bar{\sigma}_{3(n)})(u_1^2 + \delta v_1^2).
\end{aligned} \tag{20}$$

The precision of inversion of the average friction coefficient $\bar{\mu}_0$ is described by the confidence interval. Since the theoretical distribution of $\Delta(\sigma_{1(n)} - \sigma_{3(n)})$ is unknown, it is difficult to determine accurately the confidence interval for the average friction coefficient. However, the least squares method implicitly assumes that the misfit ratio $\Delta\sigma$ defined in equation (17) follows the normal distribution, and consequently the confidence interval for the average friction coefficient can be estimated based on this assumption. Since $\Delta\sigma$ can be considered as an independent random variable for each fault, using the theorem that the sum of the squares of independent normal variables follows the χ^2 distribution (Blom 1989, p. 235), we obtain

$$\frac{1}{v_{\Delta\sigma}} \sum_{i=1}^{n} \Delta\sigma_i^2 \sim \chi^2(n-1), \tag{21}$$

where $v_{\Delta\sigma}$ is the variance of $\Delta\sigma$. The estimated (or sample) variance of $\Delta\sigma$ can be expressed as

$$\hat{v}_{\Delta\sigma} = \frac{1}{(n-1)} \sum_{i=1}^{n} \Delta\sigma_i^2. \tag{22}$$

Comparing equation (22) with equation (18), it follows that the sample variance $\hat{v}_{\Delta\sigma}$ is the minimum of all possible values, each of which corresponds to a selected average friction coefficient. Therefore, choosing a confidence level $1 - \alpha_2$, the confidence interval for the inverted average friction coefficient can be estimated by constructing a one-sided confidence interval for $v_{\Delta\sigma}$. From equation (21), this is

$$v_{\Delta\sigma} < \frac{1}{\chi^2_{1-\alpha_2}(n-1)} \sum_{i=1}^{n} \Delta\sigma_i^2. \tag{23}$$

Once the one-side confidence intervals for the variance $v_{\Delta\sigma}$ are estimated, the confidence limits for the average friction coefficient $\bar{\mu}_0$ are simply those two values that yield the confidence limits for $v_{\Delta\sigma}$. Following the same procedure, the confidence intervals for the normalized three principal stresses (equation 20) can be estimated.

The only parameter which cannot be determined by the combination of mechanical and geometrical constraints is the pore fluid pressure. Equation (12) shows

Table 1. Results of stress inversion for sites AVB, TYM, KAM, and LOD with two different methods. The three principal stress axes are denoted by σ_1, σ_2 and σ_3, respectively. Stress ratio and estimated standard deviation for the measurement errors in both fault plane orientation and slip direction are denoted by δ and $\hat{v}^{1/2}$. The computing precision is 1° for σ_1, σ_2 and σ_3, 0.01 for δ and 0.1° for $\hat{v}^{1/2}$

		σ_1		σ_2		σ_3			
Site	Method	Plunge	Trend	Plunge	Trend	Plunge	Trend	δ	$\hat{v}^{1/2}$
AVB	Criterion C_6	75°	63°	15°	244°	0°	154°	0.26	7.4°
	Criterion MC_1	73°	71°	17°	238°	4°	330°		
TYM	CriterionC_6	83°	202°	6°	57°	4°	327°	0.13	12.1°
	Criterion MC_1	83°	236°	7°	53°	0°	143°		
KAM	Criterion C_6	82°	242°	3°	131°	8°	41°	0.32	20.4°
	Criterion MC_1	78°	229°	6°	108°	10°	16°		
LOD	Criterion C_6	90°	—	0°	99°	0°	189°	0.14	5.8°
	Criterion MC_1	82°	279°	8°	113°	2°	23°		

that the role played by pore fluid pressure in faulting is opposite to that of depth, that is, increasing the pore fluid pressure is equivalent to decreasing the depth. Since depth does not affect fault orientations and slip directions (for a given stress field), the pore fluid pressure is unlikely to be relevant to the geometry of faulting (fault orientation and slip direction), although it is certainly relevant to the mechanics of faulting. Consequently, the pore fluid pressure cannot be determined by a method based on inversion of fault-slip data.

EXAMPLES

The new inversion method is applied to four field examples. The first three data sets (Angelier 1990) were used by Yin & Ranalli (1993) to invert for the principal stress directions and the stress ratio. The first data set consists of 33 faults, from Neogene reef limestone near Agia Vavara, central Crete, Greece (site AVB). The second data set consists of 50 faults, from Neogene marly limestone near Tymbaki, southern Crete, Greece (site TYM). The third data set consists of 50 faults, from the Mineoka ophiolite of Kamogeawa, Boso Peninsula, Central Japan (site KAM). The fourth data set (Etchecopar *et al.* 1981) is composed of 38 faults, from the Permian basin of Lodève, Hérault, France (site LOD). These four areas are characterized by extensional tectonics with predominant normal dip-slip and oblique-slip faulting. Both the fault plane orientations and slip directions for sites KAM and LOD are more scattered than those for sites AVB and TYM [see Angelier (1990) and Etchecopar *et al.* (1981) for detailed descriptions].

In order to compare our results with the method proposed by Reches *et al.* (1992), the four data sets are also inverted by criterion MC_1 (the selected range of the average friction coefficient is 0.0–1.5, and the increment in each iterative step is 0.01 for the friction coefficient and 1° for the principal stress directions). Table 1 lists the three principal stress directions inverted by criterion C_6 (Yin & Ranalli 1993) and criterion MC_1, and the stress ratio δ inverted by criterion C_6 (because the stress ratio cannot be determined by criterion MC_1).

Reches *et al.* (1992) argued that methods based on the geometrical constraint are less powerful than criterion MC_1 because the misfit angle between the predicted maximum shear stress and slip direction may vanish for large numbers of general stress fields. For instance, when both σ_1 and σ_3-axis lie in the plane defined by the slip and the fault normal, the misfit angle between predicted shear stress and measured slip direction vanishes for $0° < \psi < 90°$. This argument is certainly valid for conjugate faults. However, faulting in anisotropic rocks is characterized by reactivation of preexisting faults. The above argument is no longer valid in this case. When the distribution of fault plane orientations diverges notably from conjugate sets, the methods based on the geometrical constraint are more powerful than criterion MC_1 in the determination of the principal stress directions. This point is well supported by the four

Table 2. Inversion results for the average friction coefficient ($\bar{\mu}_0$), average normalized critical stress difference ($\bar{\sigma}_{1(n)}$–$\bar{\sigma}_{3(n)}$), average normalized principal stresses ($\bar{\sigma}_{1(n)}, \bar{\sigma}_{2(n)}, \bar{\sigma}_{3(n)}$) for sites AVB, TYM, KAM, and LOD with two different methods

Site	Method	$\bar{\mu}_0$	$(\bar{\sigma}_{1(n)}-\bar{\sigma}_{3(n)})$	$\bar{\sigma}_{1(n)}$	$\bar{\sigma}_{2(n)}$	$\bar{\sigma}_{3(n)}$
AVB	Criterion MC_1	0.68				
	Criterion MC_3	0.64	0.78	1.04	0.45	0.26
TYM	Criterion MC_1	0.63				
	Criterion MC_3	0.70	0.78	1.01	0.33	0.23
KAM	Criterion MC_1	0.0				
	Criterion MC_3	0.22	0.46	1.01	0.70	0.55
LOD	Criterion MC_1	0.96				
	Criterion MC_3	0.88	0.89	1.00	0.23	0.11

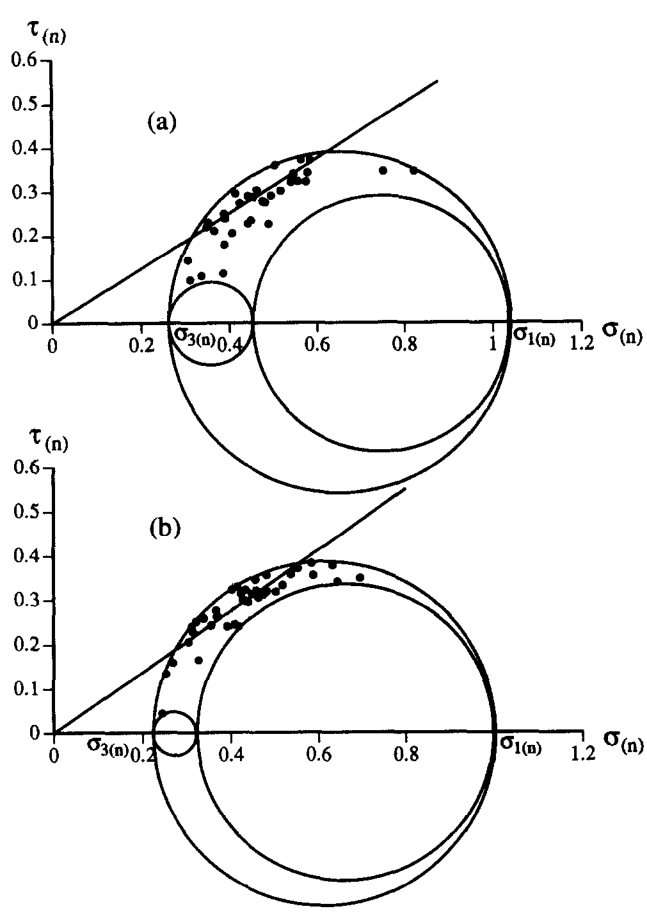

Fig. 3. Distribution of fault plane orientations (dots), inverted average friction coefficient (slope of straight line), and normalized critical stress difference (Mohr circles). (a) Site AVB; (b) site TYM.

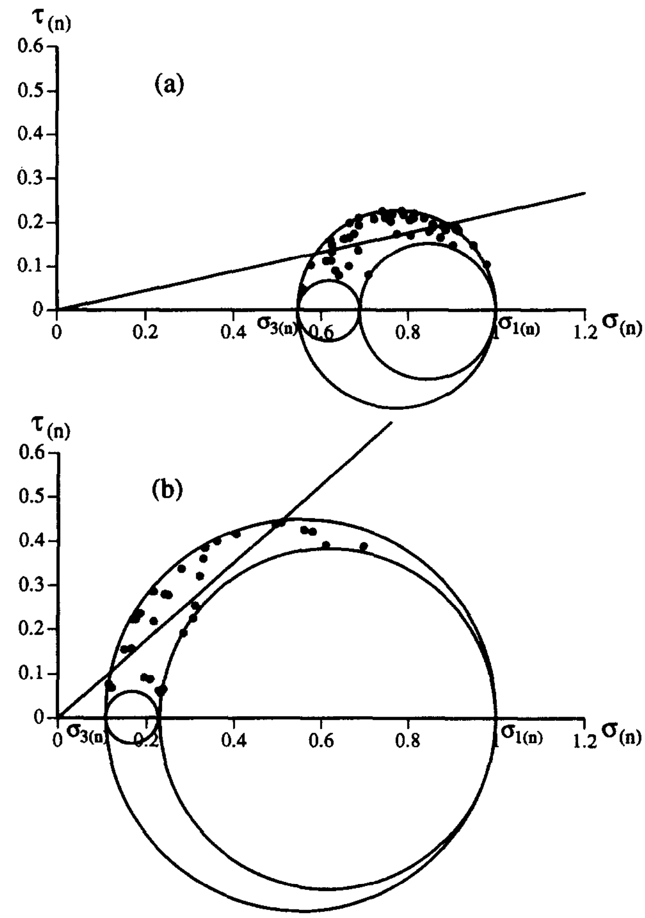

Fig. 4. Distribution of fault plane orientations (dots), inverted average friction coefficient (slope of straight line), and normalized critical stress difference (Mohr circles). (a) Site KAM; (b) site LOD.

examples studied. Comparing the results obtained by criteria C_6 and MC_1, we find that, when the distribution of both fault plane orientations and slip directions is only slightly scattered, such as in the cases of sites AVB and TYM, the two methods (criteria C_6 and MC_1) give rise to similar results. However, when the distribution of fault plane orientations and slip directions is more scattered, such as for site KAM and LOD, the difference between criteria C_6 and MC_1 becomes larger.

The misfit angle between predicted shear stress and measured slip direction, its variance, and the normalized misfit angle for each fault are calculated. The normalized misfit angles are used to test the hypothesis that the stress field is uniform for each data set, according to the χ^2 procedure described by Yin & Ranalli (1993). The significant level at which the null hypothesis is rejected is $\alpha_1 = 0.693$ for site AVB, $\alpha_1 = 0.702$ for site TYM, $\alpha_1 = 0.563$ for site KAM, and $\alpha_1 = 0.369$ for site LOD. Therefore, the hypothesis that the stress field is uniform is accepted for all the four examples at a significance level much higher than the conventional $\alpha_1 \leq 0.05$. However, the estimated standard deviation of measurement error for site KAM is unexpectedly large ($\hat{v}^{1/2} = 20.4°$). This may indicate a nonuniform stress field, because the χ^2 method tests only the shape of distribution of a random variable but not its variance (usually the estimated or sample variance is used in the χ^2 testing).

Table 2 lists the results of the inversion for the average friction coefficient of faults in the four areas according to criteria MC_1 and MC_3. The average normalized critical stress difference and average normalized magnitudes of principal stresses are calculated according to equations (14) and (20). The results obtained by criterion MC_3 are also plotted in Figs. 3 and 4. As in the case of principal stress directions, criteria MC_1 and MC_3 yield similar average friction coefficients for sites AVB and TYM, but noticeably different results for sites KAM and LOD, where the data are more dispersed. This suggests that the difference between criterion MC_3 and criterion MC_1 increases with the increase of the dispersion in fault plane orientations and slip directions, i.e. the deviation from a conjugate fault set. In three of the four examples, the new method yields an average friction coefficient ranging from 0.64 to 0.88, i.e. within the range obtained in laboratory experiments (Byerlee 1978). For site KAM, a relatively small average friction coefficient $\bar{\mu}_0 = 0.22$ is obtained. However, as mentioned above, the estimated standard deviation of measurement errors for site KAM is unexpectedly large, which may indicate a nonuniform stress field. When a population of faults which actually come from different stress fields is fitted by a single stress field, their orientations with respect to the inverted principal stress are greatly biased and some of the faults may be highly inclined to the principal stress axes. This results in a smaller average friction coefficient.

Table 3. Estimates of 90% confidence intervals for the average friction coefficient ($\bar{\mu}_0$), average normalized critical stress difference ($\bar{\sigma}_{1(n)}$–$\bar{\sigma}_{3(n)}$), and average normalized principal stresses ($\bar{\sigma}_{1(n)}$, $\bar{\sigma}_{2(n)}$, $\bar{\sigma}_{3(n)}$) obtained by criterion MC_3 (see Table 2)

Site	$\bar{\mu}_0$	($\bar{\sigma}_{1(n)}$–$\bar{\sigma}_{3(n)}$)	$\bar{\sigma}_{1(n)}$	$\bar{\sigma}_{2(n)}$	$\bar{\sigma}_{3(n)}$
AVB	0.45–0.94	0.66–0.91	1.03–1.05	0.36–0.53	0.14–0.37
TYM	0.50–1.00	0.67–0.90	1.01–1.01	0.23–0.42	0.11–0.34
KAM	0.10–0.41	0.24–0.72	1.01–1.02	0.53–0.84	0.30–0.77
LOD	0.60–1.38	0.79–1.01	1.00–1.00	0.13–0.32	−0.01–0.21

The 90% confidence intervals for the average friction coefficient, average normalized critical stress difference, and average normalized magnitudes of the principal stresses inverted by criterion MC_3 are calculated, and the results are listed in Table 3. Comparing Table 3 with Yin & Ranalli's (1993) Table 2, we find that the confidence interval for the average friction coefficient is much larger than that for the principal stress directions. This suggests that the geometry of faulting, i.e. fault plane orientations and slip directions, imposes stronger constraints on the orientation of the stress field than on the magnitude of stresses and frictional strength.

CONCLUSIONS

In this paper, we have first examined the inversion method proposed by Reches *et al.* (1992) for the determination of the average friction coefficient of faults. The applicability of this method has been shown to be restricted to conjugate faults developed in homogeneous and isotropic rocks, where fault plane orientations with respect to the maximum principal stress axis are directly related to the friction coefficient of rocks. Its applicability is problematic in the case of reactivation of pre-existing faults in anisotropic rocks.

By combining the geometrical and the mechanical constraints, a new method has been proposed to determine the average friction coefficient of faults from inversion of fault-slip data. The method is based on the assumption that the normalized critical stress difference is constant for each fault, which is consistent with the assumption that the stress field is uniform. The method has been applied to four data sets. An average friction coefficient ranging from 0.64 to 0.88 has been obtained for three of the four data sets. One data set shows a relatively small average coefficient of friction (0.22), but the quality of this value is poor because of the possible effect of a non-uniform stress field. The inversion results are coincident with the average value of friction coefficient $\mu_0 = 0.75$ obtained from laboratory experiments. These results suggest that the uncertainty in the level of effective tectonic stresses in upper crust (for instance, the controversy on whether the deviatoric stress in the middle crust is of the order of tens or hundreds of megapascals) is mainly due to the uncertainty in pore fluid pressure.

The method is subject to the usual assumption of all fault-slip inversion procedures, i.e. that the stress field is uniform. This hypothesis, however, can be tested on the basis of the geometrical constraint. Introduction of probability theory into the inversion for the average friction coefficient is necessary to make use of the mechanical constraint in hypothesis testing. The other assumption (cohensionless faults) is also routinely made, although further study is required to assess its effects. The main advantage of the present method over the previous one proposed by Reches *et al.* (1992) is that it can be applied to the reactivation of pre-existing faults in rocks with strength anisotropies. The estimated friction coefficients in the crust are well within the laboratory range.

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